Quasi-static Characteristics of Asymmetrical and Coupled Coplanar-Type Transmission Lines

TOSHIHIDE KITAZAWA AND RAJ MITTRA, FELLOW, IEEE

Abstract —**In this paper, variational expressions for the capacitances of asymmetrical and coupled coplanar-type transmission lines are derived. The methods employed are quite general, and are useful for analyzing various types of coplanar-type transmission lines, with either an isotropic or anisotropic substrate. An accurate and efficient method of calculation is provided for the variational expressions, and some numerical results are presented.**

I. Introduction

Coplanar-type transmission lines have become quite attractive from the point of view of applications in microwave and millimeter-wave integrated circuits. The symmetrical coplanar waveguide (CPW) [1]–[6], as well as other similar configurations [7], [8], have received considerable attention in the literature. Recently, the asymmetrical version of the coplanar waveguide has been introduced [9], [10] because of the additional flexibility offered by the asymmetric configuration in the design of integrated circuits. The application of coplanar-type transmission lines to directional couplers was proposed by C. P. Wen [11] to achieve better isolation characteristics. However, the analytical method for a coupled line section, which is useful for obtaining design information, was based on the zeroth-order approximation, which assumed an infinitely thick substrate [11].

This paper derives the variational expressions for the line capacitances of the asymmetrical and coupled coplanar-type transmission lines. These expressions are quite general and are applicable to a wide class of coplanar-type transmission lines, including those containing an anisotropic media. These expressions are employed in conjunction with an accurate and efficient numerical method based on the Ritz procedure. Numerical results are shown for the line characteristics of asymmetrical and coupled coplanar-type lines with anisotropic media and are compared with the exact analytical solutions for the special case of air as the substrate material. The comparison is found to be quite favorable.

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The authors are with the Department of Electrical and Computer Engineering, University of Illinois at Urbana–Champaign, Urbana, IL 61801.

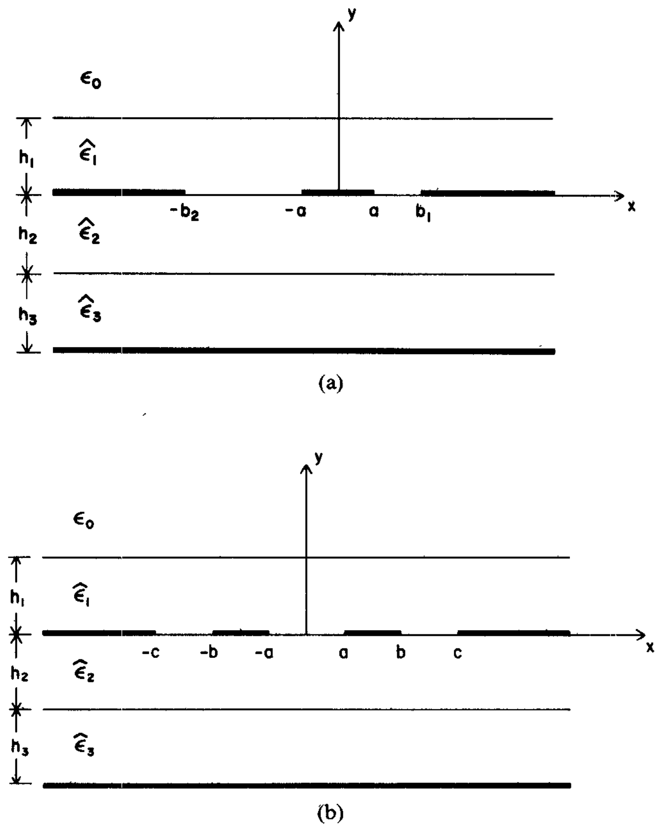


Fig. 1. (a) General structure of asymmetrical coplanar-type transmission lines.

$$W_i = b_i - a \qquad (i=1,2)$$

$$S_{\substack{1\\2}} = \pm\left(a + b_{\substack{1\\2}}\right)\Big/2.$$

(b) General structure of open coupled coplanar-type transmission lines.

II. Variational Expressions for the Line Capacitances

In this section, the formulation procedure will be outlined for the general structures of asymmetrical coplanar-type (A-CTL, Fig. 1(a)) and coupled coplanar-type transmission lines (C-CTL, Fig. 1(b)), which include various coplanar-type transmission lines in Fig. 2. The layered media in Fig. 1 are uniaxially anisotropic and their permit-

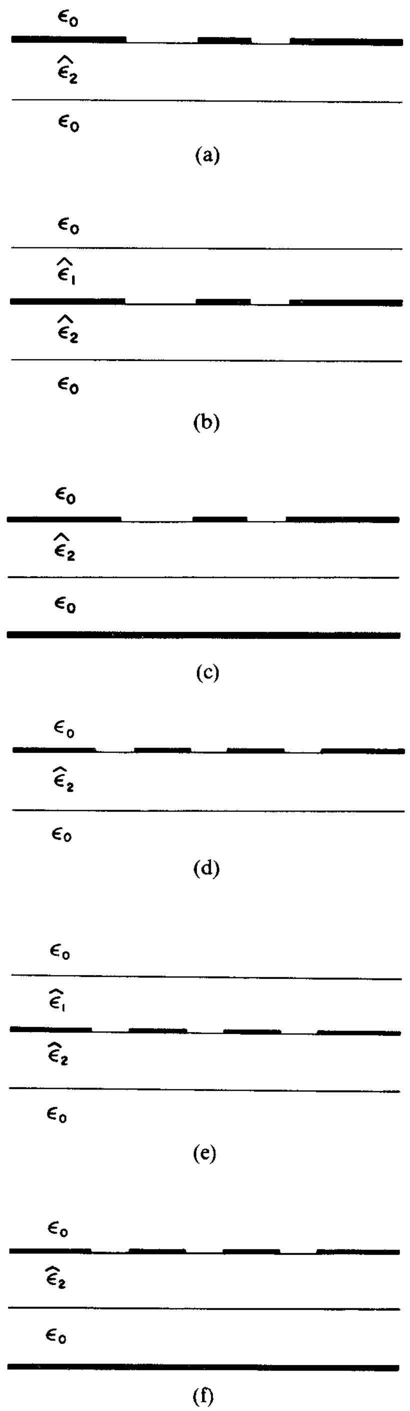


Fig. 2. (a) Asymmetrical coplanar waveguide. (b) Asymmetrical sandwich coplanar waveguide. (c) Asymmetrical conductor-backed coplanar waveguide. (d) Coupled coplanar waveguide. (e) Coupled sandwich coplanar waveguide. (f) Coupled conductor-backed coplanar waveguide.

tivities are given by the following dyadic:

$$\hat{\epsilon}_i = \begin{bmatrix} \epsilon_{ixx} & \epsilon_{ixy} \\ \epsilon_{ixy} & \epsilon_{iyy} \end{bmatrix} \epsilon_0, \qquad i = 1,2,3 \tag{1}$$

where

$$\begin{aligned} \epsilon_{ixx} &= \epsilon_{i\parallel} \cos^2\gamma_i + \epsilon_{i\perp} \sin^2\gamma_i \\ \epsilon_{iyy} &= \epsilon_{i\parallel} \sin^2\gamma_i + \epsilon_{i\perp} \cos^2\gamma_i \\ \epsilon_{ixy} &= (\epsilon_{i\parallel} - \epsilon_{i\perp}) \sin\gamma_i \cos\gamma_i \end{aligned} \tag{2}$$

γ_i is the angle of the optical axis from the x-axis, and $\epsilon_{i\parallel}$ and $\epsilon_{i\perp}$ are the relative permittivities longitudinal and transverse to the optical axis, respectively.

A. Variational Expression for the Line Capacitance of Asymmetrical Coplanar-Type Transmission Lines (A-CTL)

From a solution to the Laplace's equation, the charge distribution on the conductors at $y = 0$ can be expressed in terms of the aperture field $e_x(x)$ at $y = 0$ as

$$\sigma(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_A(\alpha; x|x') e_x(x') \, dx' \, d\alpha \tag{3}$$

where

$$P_A(\alpha; x|x') = -j\alpha F_A(\alpha) e^{j\alpha(x-x')} \tag{4}$$

with

$$F_A(\alpha) = \frac{\epsilon_0}{2\pi} \{ Y_u(\alpha) + Y_L(\alpha) \} \frac{1}{|\alpha|} \tag{5}$$

and

$$Y_U(\alpha) = \frac{1 + \epsilon_{1e} \tanh(K_1 h_1 |\alpha|)}{1 + \frac{1}{\epsilon_{1e}} \tanh(K_1 h_1 |\alpha|)} \tag{6}$$

$$Y_L(\alpha) = \frac{1 + \frac{\epsilon_{2e}}{\epsilon_{3e}} \tanh(K_2 h_2 |\alpha|) \tanh(K_3 h_3 |\alpha|)}{\frac{1}{\epsilon_{2e}} \tanh(K_2 h_2 |\alpha|) + \frac{1}{\epsilon_{3e}} \tanh(K_3 h_3 |\alpha|)}$$

$$K_i = \sqrt{\frac{\epsilon_{ixx}}{\epsilon_{iyy}} - \left(\frac{\epsilon_{ixy}}{\epsilon_{iyy}} \right)^2}$$

$$\epsilon_{ie} = \sqrt{\epsilon_{ixx} \epsilon_{iyy} - \epsilon_{ixy}^2}.$$

The total charge Q_0 on the center strip $|x| < a$ can be expressed as follows:

$$Q_0 = \int_{x_2}^{x_1} \sigma(x) \, dx \tag{7}$$

where x_1 and x_2 can be arbitrary values in the right slot $a < x_1 < b_1$ and in the left slot $-b_2 < x_2 < -a$, respectively.

Multiplying (7) by $e_x(x_1)$ and integrating over the right slot located at $a < x_1 < b_1$, we get

$$Q_0 V_0 = \int_a^{b_1} e_x(x_1) \left\{ \int_{x_2}^{x_1} \sigma(x) \, dx \right\} dx_1 \qquad (-b_2 < x_2 < -a). \tag{8a}$$

Similarly

$$-Q_0 V_0 = \int_{-b_2}^{-a} e_x(x_2) \left\{ \int_{x_2}^{x_1} \sigma(x) \, dx \right\} dx_2 \qquad (a < x_1 < b) \tag{8b}$$

where V_0 is the potential difference between the center strip and the ground conductors, i.e.,

$$V_0 = \int_a^{b_1} e_x(x) \, dx = -\int_{-b_2}^{-a} e_x(x) \, dx. \tag{9}$$

Substituting (3) into (8), subtracting (8b) from (8a), and rearranging the resulting expression, we obtain the line capacitance

$$C=\frac{Q_0}{V_0}=\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}e_x(x)G_A(\alpha;x|x')e_x(x')\,d\alpha\,dx'\,dx}{\left\{\int_a^{b_1}e_x(x)\,dx\right\}^2} \tag{10}$$

where the Green's function G_A is given by

$$G_A(\alpha;x|x')=2F_A(\alpha)\cos\{\alpha(x'-x)\}. \tag{11}$$

As expected, (10) reduces to [5, eq. (2)] for the symmetrical coplanar waveguide. It can be verified that (10) has the stationary property, and the value of the capacitance derived from this expression will always be larger than the true value, that is, the upper bound. Also, (10), (11), and (5) show the relationship between the anisotropic ($\epsilon_{i\perp}, \epsilon_{i\parallel}, h_i$) and the isotropic ($\epsilon_{ie}, K_i h_i$) cases. A similar connection between the two types of problems has been observed elsewhere [5], [12]–[18] for other configurations.

B. Variational Expression for the Line Capacitance of Coupled Coplanar-Type Transmission Lines (C-CTL)

The charge distribution on the conductors at $y=0$ can be related to the aperture field $e_x(x)$ at $y=0$ plane by using a procedure similar to that used for the asymmetrical coplanar-type transmission line. The charge distribution $\sigma(x)$ is given by

$$\sigma(x)=\int_0^{\infty}\int_0^{\infty}P_C(\alpha;x|x')e_x(x')\,dx'\,d\alpha \tag{12}$$

where

$$\begin{aligned}P_C(\alpha;x|x')&=-\alpha F_C(\alpha)\cos(\alpha x)\sin(\alpha x')\\&\quad\text{(for the even mode)}\\&=-\alpha F_C(\alpha)\sin(\alpha x)\cos(\alpha x')\\&\quad\text{(for the odd mode)}\end{aligned} \tag{13}$$

with

$$F_C(\alpha)=\frac{2\epsilon_0}{\pi}\{Y_U(\alpha)+Y_L(\alpha)\}\frac{1}{\alpha}. \tag{14}$$

The symmetry of the geometry with respect to the plane $x=0$ has been utilized in writing (12). The odd mode is considered in what follows, and a perfect electric conductor plane may be placed at the $x=0$ plane in this case.

The total charge located between x_1 and x_2 is given by

$$Q(x_1,x_2)=\int_{x_1}^{x_2}\sigma(x)\,dx. \tag{15}$$

When x_1 lies within the inner slot $0<x_1<a$ and x_2 lies within the outer slot $b<x_2<c$, then $Q(x_1,x_2)$ is equal to a constant Q_0, that is, the total charge on the strip $a<x<b$

$$\begin{aligned}Q_0&=Q(x_1,x_2)\\&=\int_0^{\infty}\int_0^{\infty}F_c(\alpha)\{\cos(\alpha x_2)-\cos(\alpha x_1)\}\\&\quad\times\cos(\alpha x')e_x(x')\,dx'\,d\alpha,\\&\qquad 0<x_1<a \text{ and } b<x_2<c.\end{aligned} \tag{16}$$

Multiplying (16) by $e_x(x_1)$ and integrating over $0<x_1<a$, we obtain

$$\begin{aligned}Q_0\int_0^a e_x(x_1)\,dx_1&=-Q_0V_0\\&=\int_0^{\infty}\int_0^{\infty}F_C(\alpha)\left\{-V_0\cos(\alpha x_2)-\int_0^a e_x(x_1)\cos(\alpha x_1)\,dx_1\right\}\cos(\alpha x')e_x(x')\,dx'\,d\alpha \qquad (b<x_2<c).\end{aligned} \tag{17}$$

Similarly, we can obtain

$$\begin{aligned}Q_0\int_b^c e_x(x_2)\,dx_2&=Q_0V_0\\&=\int_0^{\infty}\int_0^{\infty}F_C(\alpha)\left\{\int_b^c e_x(x_2)\cos(\alpha x_2)\,dx_2-V_0\cos(\alpha x_1)\right\}\cos(\alpha x')e_x(x')\,dx'\,d\alpha \qquad (0<x_1<a)\end{aligned} \tag{18}$$

where V_0 is the potential difference between the strip conductor $a<x<b$ and the ground conductor $c<x$. V_0 is given by

$$V_0=\int_b^c e_x(x)\,dx=-\int_0^a e_x(x)\,dx. \tag{19}$$

Subtracting (17) from (18) and utilizing (16), we obtain

$$\begin{aligned}Q_0V_0&=\int_0^a\int_0^{\infty}\int_0^{\infty}F_C(\alpha)\cos(\alpha x_1)\cos(\alpha x')\\&\quad\cdot e_x(x_1)e_x(x')\,dx'\,d\alpha\,dx_1\\&\quad+\int_b^c\int_0^{\infty}\int_0^{\infty}F_C(\alpha)\cos(\alpha x_2)\cos(\alpha x')\\&\quad\cdot e_x(x_2)e_x(x')\,dx'\,d\alpha\,dx_2.\end{aligned} \tag{20}$$

Therefore, the line capacitance of the odd mode can be expressed as follows:

$$C=\frac{Q_0}{V_0}=\frac{\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}e_x(x)G^0(\alpha;x|x')e_x(x')\,d\alpha\,dx'\,dx}{\left\{\int_b^c e_x(x)\,dx\right\}^2} \tag{21}$$

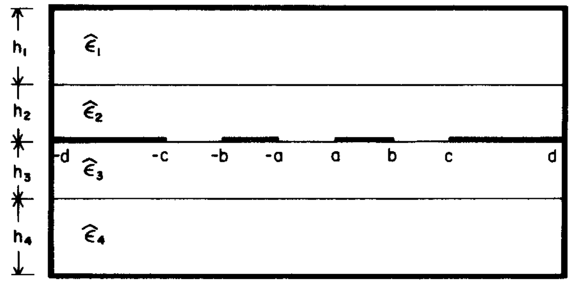


Fig. 3. Shielded coupled coplanar-type transmission lines.

where Green's function $G^0(\alpha; x|x')$ is given by

$$G^0(\alpha; x|x') = F_C(\alpha)\cos(\alpha x)\cos(\alpha x'). \quad (22)$$

For the even mode, a similar expression can be obtained by following the same procedure. The Green's function G^E for the even mode is given by

$$G^E(\alpha; x|x') = F_C(\alpha)\sin(\alpha x)\sin(\alpha x'). \quad (23)$$

Equation (21) is stationary and provides an upper bound to the line capacitance C. The transition from the anisotropic case to the equivalent isotropic case can be verified in the same manner as for the asymmetrical case.

The procedure described above can be applied to a shielded coupled coplanar-type transmission line (Fig. 3) which is filled either with an isotropic or an anisotropic medium whose optical axis coincides with one of the coordinate axes. The line capacitances for the even and odd modes are given by

$$C = \frac{\sum_n \int_0^d \int_0^d e_x(x) G_n^l(x|x') e_x(x')\, dx'\, dx}{\left\{\int_b^c e_x(x)\, dx\right\}^2}, \qquad l = E, O \quad (24)$$

where

$$G_n^E(x|x') = F_n \sin(\alpha_n x)\sin(\alpha_n x'), \qquad \text{even mode}$$
$$G_n^0(x|x') = F_n \cos(\alpha_n x)\cos(\alpha_n x'), \qquad \text{odd mode} \quad (25)$$

with

$$F_n = \frac{\eta_n \epsilon_0}{d}(Y_{Un} + Y_{Ln})\frac{1}{\alpha_n}$$

and

$$Y_{Un} = \frac{1 + \frac{\epsilon_{2e}}{\epsilon_{1e}}\tanh(K_1 h_1 \alpha_n)\tanh(K_2 h_2 \alpha_n)}{\frac{1}{\epsilon_{1e}}\tanh(K_1 h_1 \alpha_n) + \frac{1}{\epsilon_{2e}}\tanh(K_2 h_2 \alpha_n)}$$

$$Y_{Ln} = \frac{1 + \frac{\epsilon_{3e}}{\epsilon_{4e}}\tanh(K_3 h_3 \alpha_n)\tanh(K_4 h_4 \alpha_n)}{\frac{1}{\epsilon_{3e}}\tanh(K_3 h_3 \alpha_n) + \frac{1}{\epsilon_{4e}}\tanh(K_4 h_4 \alpha_n)}$$

$$\alpha_n = \frac{n\pi}{2d}, \qquad \eta_n = \begin{cases} 1 & (n = 0) \\ 2 & (n \neq 0) \end{cases}$$
$$n = 1, 3, 5, \cdots \text{ (even mode)}$$
$$= 0, 2, 4, \cdots \text{ (odd mode)}. \quad (26)$$

However, unlike the open waveguide problem, this method is not applicable to the shielded case filled with an anisotropic medium, whose optical axis is inclined, because the Green's functions for the latter case cannot be expressed in the form given in (25) [12].

III. Numerical Computation and Results

Numerical computations have been carried out by applying the Ritz procedure to the variational expressions (10) and (21). In this procedure, the unknown aperture fields $e_x(x)$ are expanded in terms of the appropriate basis functions as follows. For asymmetrical coplanar-type (A-CTL) transmission lines, the basis functions are

$$e_x(x) = \sum_{k=1}^{N_1} A_k^{(1)} f_k^{(1)}(x) + \sum_{k=1}^{N_2} A_k^{(2)} f_k^{(2)}(x) \quad (27a)$$

and for the coupled coplanar-type transmission lines (C-CTL), the corresponding functions are given by

$$e_x(x) = \sum_{k=1}^{N_1} B_k^{(1)} g_k^{(1)}(x) + \sum_{k=1}^{N_2} B_k^{(2)} g_k^{(2)}(x) \quad (27b)$$

where $A_k^{(i)}$ and $B_k^{(i)}$ are variational parameters which are determined such that the best approximation is obtained under the conditions of (9) and (19). The following basis functions are adopted by taking the edge effect into consideration:

$$f_k^{(i)}(x) = \frac{T_{k-1}\left\{\frac{2(x - S_i)}{W_i}\right\}}{\sqrt{1 - \left\{\frac{2(x - S_i)}{W_i}\right\}^2}}, \qquad i = 1,2$$

$$S_{\substack{1\\2}} = \pm\left(a + b_{\substack{1\\2}}\right)\Big/2, \qquad W_{\substack{1\\2}} = b_{\substack{1\\2}} - a \quad (28)$$

$$g_k^{(1)} = \frac{T_{N(k)}\left(\frac{x}{a}\right)}{\sqrt{1 - \left(\frac{x}{a}\right)^2}} \quad (29)$$

$$g_k^{(2)} = \frac{T_{k-1}\left\{\frac{2(x - S)}{W}\right\}}{\sqrt{1 - \left\{\frac{2(x - S)}{W}\right\}^2}},$$

$$W = c - b, \qquad S = (b + c)/2 \quad (30)$$

where $T_k(y)$ is Chebyshev's polynomial of the first kind and the suffix $N(k)$ in (29) is given by

$$N(k) = 2k - 1 \qquad \text{(even mode)}$$
$$= 2(k - 1) \qquad \text{(odd mode)}.$$

The accuracy of the computation depends on the number of basic functions, i.e., N_1 and N_2 in (27). Tables I and II show the preliminary numerical results of the normalized capacitance C/ϵ_0 of the A-CTL and open C-CTL for different values of N_1, N_2. Special cases with $\epsilon_{t\parallel} = \epsilon_{t\perp} = 1$ and $h_3 \to \infty$ are considered in Tables I and II, for which the exact analytical solutions can be obtained by conformal

TABLE I
NORMALIZED LINE CAPACITANCE C/ϵ_0 OF ASYMMETRICAL COPLANAR-TYPE TRANSMISSION LINE

a/W_1	W_2/W_1	N_1 1, N_2 1	N_1 2, N_2 2	N_1 3, N_2 3	Conformal mapping
	1	2.199	2.107	2.105	2.105
0.25	2	2.085	1.946	1.940	1.940
	4	2.085	1.858	1.838	1.836
	1	3.520	3.510	3.510	3.510
1.50	2	3.219	3.198	3.198	3.198
	4	3.005	2.956	2.956	2.956

$\epsilon_{i\parallel} = \epsilon_{i\perp} = 1 \quad (i = 1,2,3),\ h_3 \to 0.$

TABLE II
NORMALIZED LINE CAPACITANCE C/ϵ_0 OF OPEN COUPLED COPLANAR-TYPE TRANSMISSION LINES

Even Mode

$\frac{c-b}{b-a}$	$\frac{2a}{(b-a)}$ \ N_1, N_2	N_1 1, N_2 2	N_1 2, N_2 3	Conformal mapping
0.5	0.2	1.9469	1.9469	1.9469
	1.0	2.0620	2.0619	2.0619
2.0	0.2	1.3130	1.3128	1.3128
	1.0	1.4043	1.4042	1.4041

Odd Mode

$\frac{c-b}{b-a}$	$\frac{2a}{(b-a)}$ \ N_1, N_2	N_1 2, N_2 2	N_1 3, N_2 3	Conformal mapping
0.5	0.2	5.2218	5.2217	5.2217
	1.0	3.6322	3.6322	3.6322
2.0	0.2	4.9089	4.9012	4.9009
	1.0	3.2537	3.2495	3.2493

$\epsilon_{i\parallel} = \epsilon_{i\perp} = 1 \quad (i = 1,2,3),\ h_3 \to 0.$

mapping (Appendix I). It should be noted that accurate results are obtained by using a small number of suitably chosen basis functions for a wide range of parameters. For instance, for the calculations presented herein, $N_1 = N_2 = 2$ was used for A-CTL. For C-CTL, $N_1 = 1$, $N_2 = 2$ for even modes and $N_1 = N_2 = 2$ for odd modes have been employed.

Figs. 4 and 5 present numerical examples for the asymmetrical coplanar waveguide in Fig. 2(a). Fig. 4 shows the effective dielectric constant ϵ_{eff} and the characteristic impedance Z_0 as a function of the width ratio W_2/W_1 for an asymmetrical coplanar waveguide with an isotropic dielectric substrate. The values for the symmetrical case ($W_2/W_1 = 1$) [4] are included only to indicate the accuracy of the computation. The effective dielectric constant ϵ_{eff} becomes smaller and the characteristic impedance Z_0 becomes larger as the width ratio W_2/W_1 increases.

Fig. 5 shows the results for an asymmetrical coplanar waveguide on an anisotropic sapphire substrate. The effective dielectric constant and the characteristic impedance are shown as a function of the inclination of the optical axis γ_2.

In Fig. 6, the even- and odd-mode characteristics of open C-CTL, shown in Fig. 2(d), are compared with those based on the zeroth-order approximation as presented in [11]. The error in the zeroth-order solution becomes larger for a thinner substrate because the substrate thickness is assumed to be infinite in this solution.

Fig. 7 shows the even- and odd-mode characteristics of the open C-CTL on an isotropic substrate as a function of electrode configuration. The dependence on the width of inner slot $2a$ is less for even modes; in contrast, the dependence on the width of the outer slot $(C-b)$ is less for odd modes.

Fig. 8 shows the characteristics of the open C-CTL on an anisotropic substrate ($\epsilon_{2\parallel} = 5.12$, $\epsilon_{2\perp} = 3.40$). The effective dielectric constant ϵ_{eff} and the characteristic impedance Z_0 are shown as a function of the angle γ_2 of the optical axis from the x-axis.

Fig. 9 shows the results for a shielded C-CTL with an anisotropic sapphire substrate ($\epsilon_{3\parallel} = 11.6$, $\epsilon_{3\perp} = 9.4$, $\gamma_3 = \pi/2$). It is noted that the even modes are more sensitive than the odd modes to the variation in h_4, the spacing between the substrate and the shielding conductor. Equal even- and odd-mode phase velocities are obtained by adjusting h_4, and under these conditions high directivity can be achieved in a directional coupler design that employs these lines.

IV. CONCLUSIONS

In this paper, variational expressions for the line capacitances are derived for the general structures of asymmetrical and coupled coplanar-type transmission lines. The present analysis, which is valid for anisotropic media, suggests a relationship between the anisotropic and the equivalent isotropic case.

An efficient computational scheme, based on the Ritz procedure, has been employed for the numerical computations. Numerical results have been compared with the exact analytical solutions for the special case of air as the substrate material; excellent agreement has been found for a wide range of parameters. Some numerical data for asymmetrical and coupled coplanar-type transmission lines are shown for both the isotropic and anisotropic substrates.

APPENDIX I
ASYMMETRICAL COPLANAR WAVEGUIDE WITHOUT SUBSTRATES

The line capacitances of the asymmetrical coplanar waveguide without substrates can be evaluated analytically by a repeated application of conformal mapping. A series of transformations are shown in Fig. 10. The determinantal

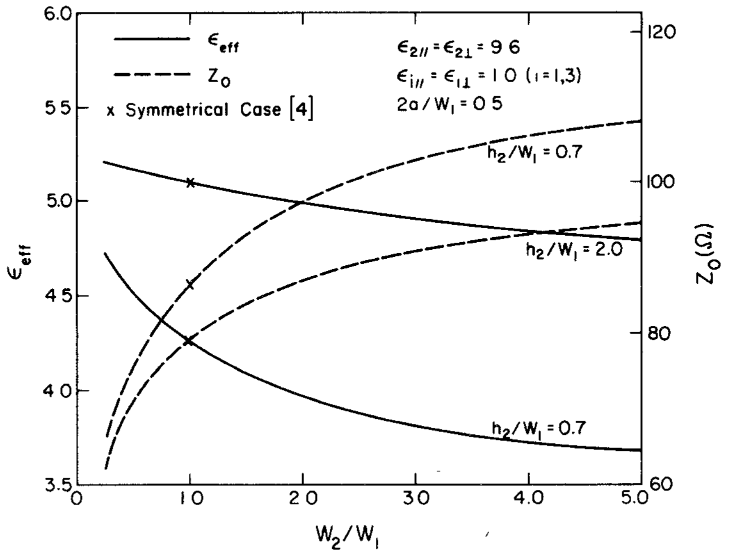


Fig. 4. Asymmetrical coplanar waveguide on isotropic dielectric substrate.

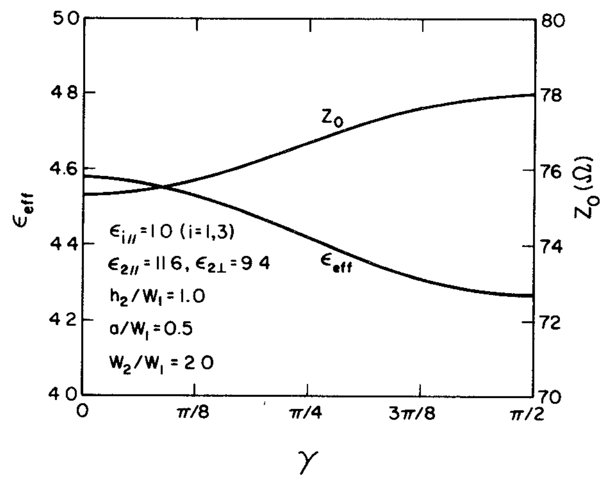


Fig. 5. Asymmetrical coplanar waveguide on anisotropic sapphire substrate.

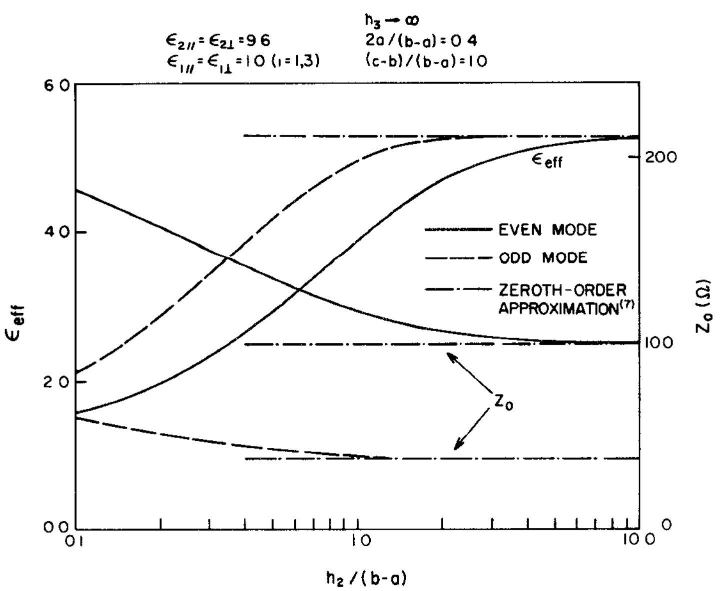
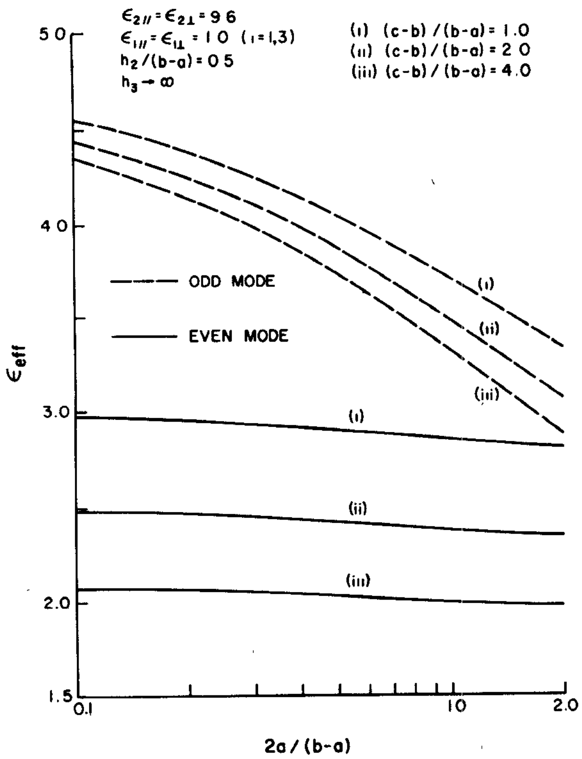
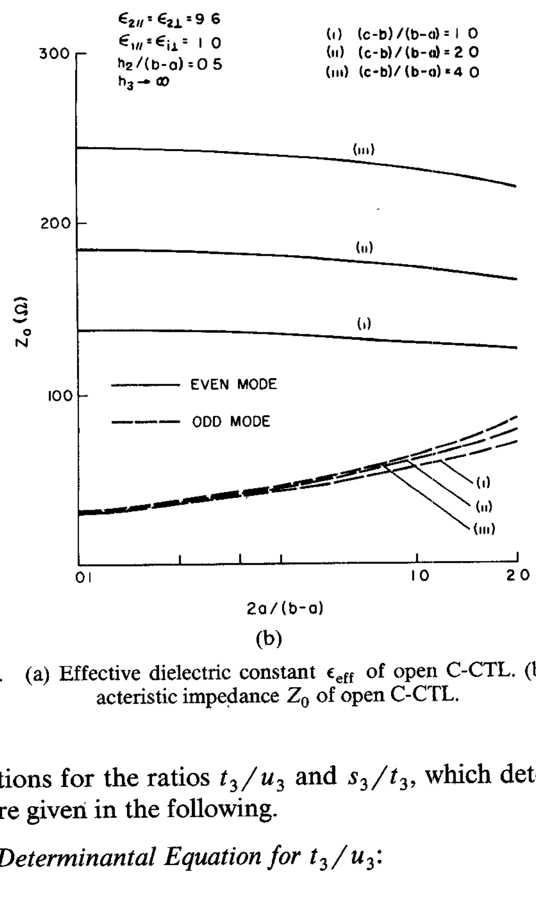


Fig. 6. The comparison of this method with the zeroth-order approximation of C-CTL.



(a)



(b)

Fig. 7. (a) Effective dielectric constant ϵ_{eff} of open C-CTL. (b) Characteristic impedance Z_0 of open C-CTL.

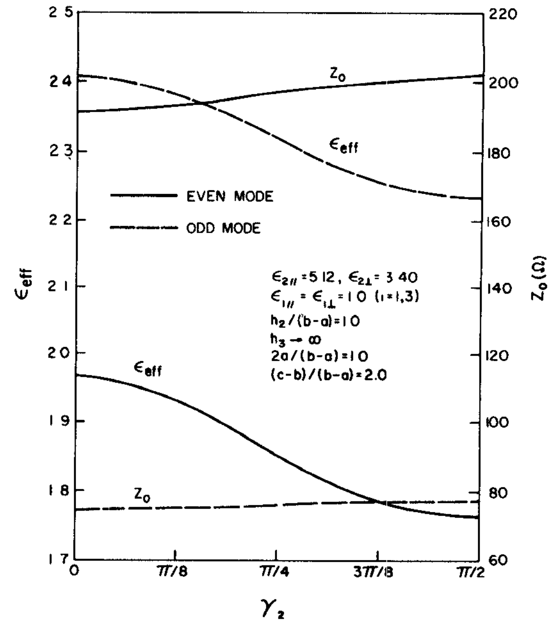


Fig. 8. Effective dielectric constant and characteristic impedance of C-CTL versus γ_2.

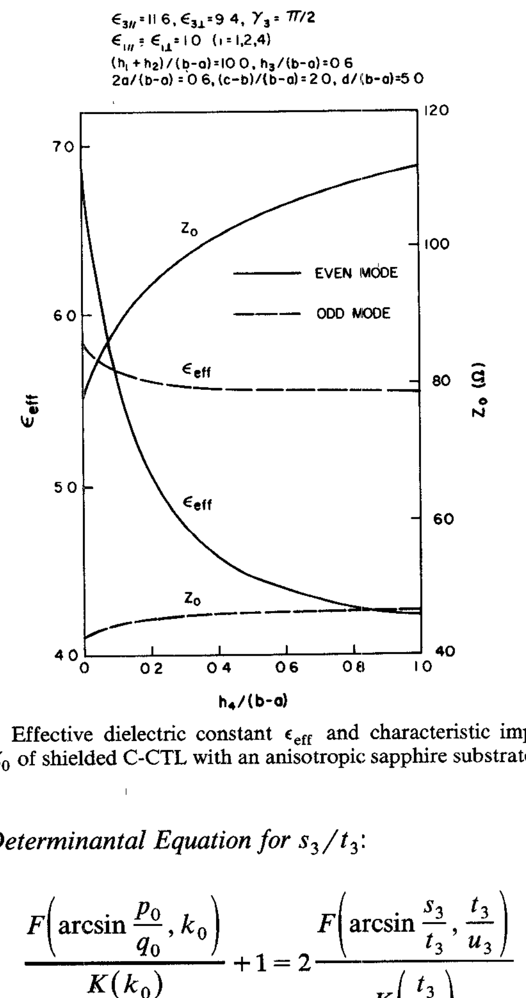


Fig. 9. Effective dielectric constant ϵ_{eff} and characteristic impedance Z_0 of shielded C-CTL with an anisotropic sapphire substrate.

equations for the ratios t_3/u_3 and s_3/t_3, which determine k_3, are given in the following.

The Determinantal Equation for t_3/u_3:

$$2\frac{K(k_0)}{K'(k_0)} = \frac{K\left(\frac{t_3}{u_3}\right)}{K'\left(\frac{t_3}{u_3}\right)}. \tag{A1}$$

The Determinantal Equation for s_3/t_3:

$$\frac{F\left(\arcsin\frac{p_0}{q_0}, k_0\right)}{K(k_0)} + 1 = 2\frac{F\left(\arcsin\frac{s_3}{t_3}, \frac{t_3}{u_3}\right)}{K\left(\frac{t_3}{u_3}\right)} \tag{A2}$$

where $F(a, b)$ is the elliptic integral of the first kind and $K(k)$ is the complete elliptic integral of the first kind.

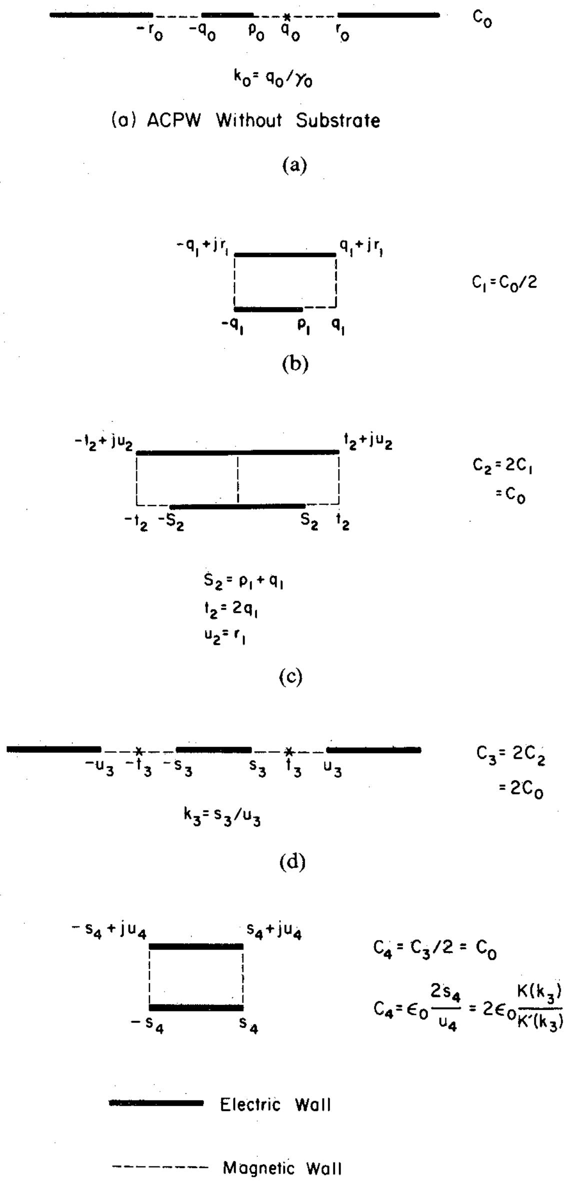


Fig. 10. A series of transformations for the asymmetrical coplanar waveguide without substrates.

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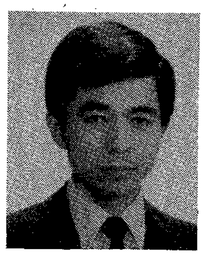

Toshihide Kitazawa was born in Sapporo, Japan, on December 1, 1949. He received the B.E., M.E., and D.E. degrees in electronics engineering from Hokkaido University, Sapporo, Japan, in 1972, 1974, and 1977, respectively.

He was a Post-Doctoral Fellow of the Japan Society for the Promotion of Science from 1979 to 1980. Since 1980, he has been an Associate Professor of Electronic Engineering at the Kitami Institute of Technology, Kitami, Japan. Currently, he is a Visiting Assistant Professor of Electrical Engineering at the University of Illinois, Urbana.

Dr. Kitazawa is a member of the Institute of Electronics and Communication Engineers of Japan.

Raj Mittra (S'54–M'57–SM'69–F'71) is the Director of the Electromagnetic Communication Laboratory of the Electrical and Computer Engineering Department and Research Professor of the Coordinated Science Laboratory at the University of Illinois. He is a Past-President of AP-S. He serves as a consultant to several industrial and governmental organizations in the United States and abroad.

His professional interests include the areas of analytical and computer-aided electromagnetics, high-speed digital circuits, radar scattering, satellite antennas, microwave and millimeter-wave integrated circuits, frequency selective surfaces, EMP and EMC analysis, and interaction of electromagnetic waves with biological media.